Econ 802

First Midterm Exam

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All questions have equal weight. If something is unclear, please ask. You may want to work first on the questions where you feel most confident.

1. Here are some questions about technology.

(a) Assume the production possibilities set Y is non-empty and all prices are positive. Explain why the firm's profit maximization problem definitely has a solution if Y is closed and bounded. Then use graphs to show why there may not be a solution if (i) Y is not closed or (ii) Y is not bounded.

(b) Consider the Cobb-Douglas production function $y = x_1^a x_2^b$ with $a > 0$ and $b > 0$. Show that the input requirement sets V(y) are always convex but the production possibilities set Y may <u>not</u> be convex. Use a combination of graphs and algebra.

(c) A firm has a single output $y \geq 0$ and an input vector $x = (x_1 \, . \, . \, x_n) \geq 0$. All prices are positive. Show that when the production function f(x) is strictly concave, the problem max $\{pf(x) - wx\}$ <u>cannot</u> have two solutions x^* and x^{**} with $x^* \neq x^{**}$.

2. Consider a firm with the production function $y = x^a$ where $x \geq 0$ is a scalar and $0 < a < 1$. Let the output price be $p > 0$ and let the (scalar) input price be $w > 0$.

(a) Compute the profit function $\pi(p, w)$.

(b) For the result in (a), it can be shown that $p \, \partial\pi(p,w)/\partial p + w \, \partial\pi(p,w)/\partial w = \pi(p, w)$. You don't need to prove this, but carefully explain why it makes economic sense.

(c) Prove that if p rises, both x and y increase, while if w rises, both x and y decrease.

3. Here are some questions about costs.

(a) Suppose $y = ax_1 + bx_2$ with $a > 0$ and $b > 0$. Prove that the cost function is $c(w, y) = y \min \{w_1/a \, ; \, w_2/b\}$.

(b) Suppose $y = (x_1^r + x_2^r)^{1/r}$ where $r < +1$, and let there be fixed input prices $(w_1, w_2) > 0$. Define the expansion path to be the set of input bundles $x = (x_1, x_2)$ that are cost minimizing at some output level $y > 0$. Draw a graph of the EP and justify your answer mathematically (hint: use the FOC and don't worry about the SOC).

(c) Suppose $y = x_1x_2$ where $x_1 \geq 0$ is variable in the short run and $x_2 > 0$ is fixed in the short run. The input prices are $(w_1, w_2) > 0$. Write out algebraic expressions for average fixed cost, average variable cost, average total cost, and marginal cost, and show these curves on a graph. Can the firm's short run profit maximization problem have a solution? What about the long run problem? Explain.

4. A firm uses two inputs and always produces the same output $y > 0$. In period s, the input price vector is w^s and the input quantity vector is x^s. In period t, these are instead w^t and x^t. The firm's behavior is consistent with cost minimization.

(a) Let VI(y) be the <u>smallest</u> input requirement set that is convex, monotonic, closed, and consistent with the data. Let QI(y) be the corresponding isoquant set. Show these sets on a graph and explain your reasoning.

(b) Let VO(y) be the <u>largest</u> input requirement set that is convex, monotonic, closed, and consistent with the data. Let QO(y) be the corresponding isoquant set. Show these sets on a graph and explain your reasoning.

(c) Assuming the true input requirement set is VI(y), use a graph to describe how the firm's behavior changes as the ratio of its input prices changes. Then do the same for VO(y). Explain your reasoning in each case.

5. Here are some miscellaneous questions.

(a) When defining the elasticity of substitution, we often differentiate the ratio x_1/x_2 with respect to the price ratio w_1/w_2. A clever undergraduate wants to know why this is valid. She points out that we normally write conditional input demands in the form $x_1(w_1, w_2, y)$ and $x_2(w_1, w_2, y)$ where these are functions of the <u>levels</u> of the prices, not the <u>ratio</u> w_1/w_2. Furthermore, the input demand functions involve the <u>levels</u> x_1 and x_2, not the <u>ratio</u> x_1/x_2. Assume that the student knows about the technical rate of substitution, and explain to her why the definition makes sense.

(b) Define the local elasticity of output with respect to scale e(x). Then show that for any production function homogenous of degree $k > 0$, it is true globally that $e(x) = k$. Finally, draw three graphs showing the general shapes of the long run average and marginal cost curves for $k < 1$, $k = 1$, and $k > 1$. Briefly justify your answers.

(c) Fix the output y as well as all prices w_j with $j \neq i$. We are only concerned with changes in the price w_i for input i. Express the cost function in the form $c(w, y) = wx(w_i) - \lambda(w_i)[f(x(w_i)) - y]$ where $x(w_i)$ is the vector of optimal inputs and $\lambda(w_i)$ is the Lagrange multiplier. Differentiate this expression with respect to w_i and then show that $\partial c(w, y)/\partial w_i = x_i(w_i)$. Give an economic interpretation for this result.